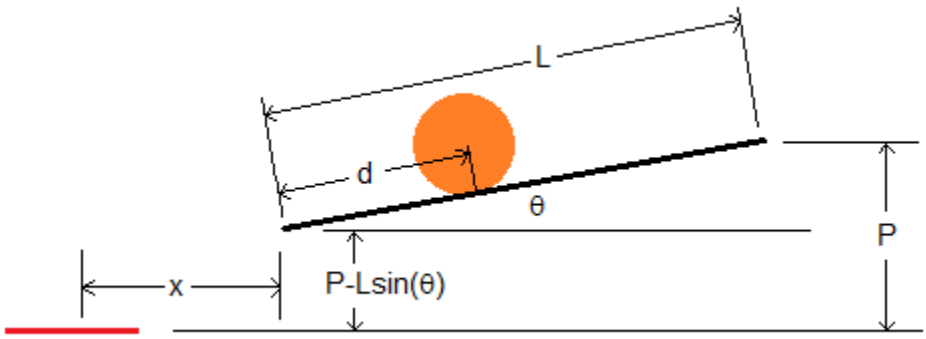
Optimization Problem for the Dumper Angle

Determine the optimum angle for the dumper surface when the lift has reached its maximum height. The optimum angle will maximize the distance, x, that the ball travels before the bottom of the ball falls to the height of the middle goal.

Assume the dumper surface is flat, not curved.

The dumper surface has a length L and when the scissor lift is at maximum extension, the dumper surface is at an angle θ from horizontal. The maximum height of any point on the dumper is P inches above the middle rim. The distance between the contact point of the first ball and the end of the dumper is d.



<u>Equations</u>

Conservation of Energy for a Rolling Ball

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

Moment of Inertia for a Sphere

$$I = \frac{2}{5}mr^2$$

Relationship between linear and rotational velocity, rolling without slipping

$$v = r\omega$$

Trigonometric relationships

$$h = \mathrm{d} \cdot \sin\theta$$

$$y = falling\ distance = P - L \cdot \sin\theta - r(1 - \cos\theta)$$

This includes a correction to account for the ball extending slightly beyond and below the end of the ramp.

Projectile motion

$$y = x\tan\theta - x^2\frac{g}{2v^2\cos^2\theta}$$

Manipulation of the equations

Energy equation

$$\frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mr^2\omega^2 = mgh$$

$$\frac{1}{2}mv^2 + \frac{1}{5}mv^2 = mgh$$

$$\frac{7}{10}v^2 = gh$$

$$\frac{7}{10}v^2 = gd\sin\theta$$

$$v = \sqrt{\frac{10}{7}gd\sin\theta}$$

Quadratic formula to find the smaller of the two x values

$$-x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Substituting $a = g/(2v^2\cos^2\theta)$, $b = -\tan\theta$, and $c = -(P - L\sin\theta)$:

$$x = \frac{-\tan\theta + \sqrt{\tan^2\theta + 2g\,y/(v\cos\theta)^2}}{g/(v\cos\theta)^2} - r\sin\theta$$

This includes a correction for the ball extending off the ramp before leaving the ramp.

By varying θ, we can find the maximum x for particular values of P, L and d.

Assuming L = 30 inches, P = 18 inches, d = 10 inches and using g of 32.2*12 = 386.4 inches/s^2 yields a maximum x value of 8.4 inches at 14 degrees.